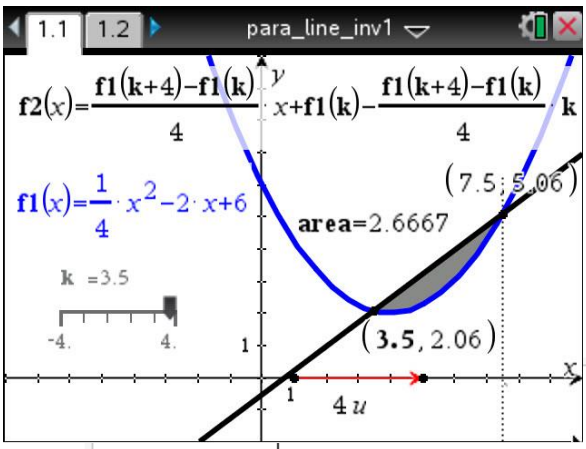
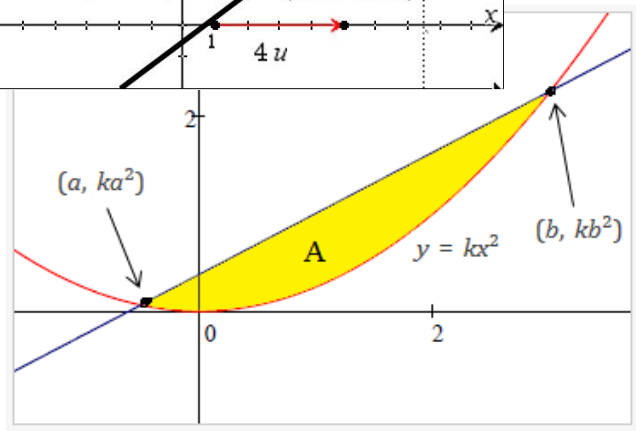
IB Mathematics Analysis & Approaches HL
Internal Assessment – The Exploration

Student Guide

1. intro / prep — **Summer 2025**
2. topic choice — **due August 11, 2025** (see last page)
3. complete draft — **due December 1, 2025**
4. teacher feedback — **by January 5, 2026**
5. final version

Final Draft due February 17, 2026





Now I will find another formula for
line pass through the points (a, ka^2) and $(b$
secant line form a to b minus the area und

(i) $y - ka^2 = \frac{kb^2 - ka^2}{b - a}(x - a)$

(ii) $y - ka^2 = \frac{k(b + a)(b - a)}{(b - a)}(x - a)$

(iii) $y = k(a + b)x - ka(a + b) + ka^2$

(iv) $y = k(a + b)x - kab$

To find the area enclosed by the secant and the parabola, I can use integration

(i) $A = \int_a^b k(a + b)x - kab - kx^2 \, dx = \left[\frac{k(a + b)x^2}{2} - kabx - \frac{kx^3}{3}\right]_a^b$

1. What is Internal Assessment (IA) in IB Mathematics ?

Internal Assessment in the **Analysis & Approaches** course and in the **Applications & Interpretation** course consists of a single internally assessed component (i.e. marked by the teacher) called a mathematical exploration (or just the "Exploration"). The Exploration contributes **20%** to your overall IB score for the course. (and 20% of your overall grade in the class)

2. What is the Exploration ?

Your Exploration is a written paper (12-20 pages) involving a mathematical topic that interests you. You will choose a topic in consultation with your teacher after conducting your own research.

3. How is the Exploration assessed ?

Your Exploration will earn a score out of 20 marks based on the following five criteria. Further details for each criterion and guidance for addressing them is provided later in this guide.

Criterion **A**	max **4** marks	**Presentation**
Criterion **B**	max 4 marks	**Mathematical Communication**
Criterion **C**	max 3 marks	**Personal Engagement**
Criterion **D**	max **3** marks	**Reflection**
Criterion **E**	max **6** marks	**Use of Mathematics**

Some important points to consider:

- In your Exploration you need to write ***about mathematics*** and not just ***do mathematics***.
- Any idea, method, content, image, etc that is not your own must be **cited** at the point in the Exploration where it is used. Just listing your sources in a bibliography is not enough and may lead to the IB deciding that malpractice has occurred.
- The Exploration is an opportunity for you to learn more about a mathematical topic in which you are genuinely interested. You will be rewarded (**personal engagement**) for explaining your interest in the topic, and for demonstrating curiosity, creativity & independent thinking.
- Your **audience is your fellow students** – that is, you need to write your Exploration so that classmates in your IB maths course can read and understand it. Thus, it is not necessary to explain basic mathematics with which your classmates will be familiar.
- You will be rewarded (**reflection**) for expressing what you think about the mathematics you are exploring and any results you obtain. You should endeavour to pose your own questions and try to answer them using a sufficient level of mathematical ideas and procedures.
- You are required to submit a **complete draft** of your Exploration – containing an introduction, a clearly stated aim (objective), a conclusion and sufficient content to address all five criteria. You will receive written feedback and then have an opportunity to revise it to submit a **final version**.
- All of the work you do on your Exploration must be your own. When finished with your **final version** you will be required to sign a 'declaration' that states, "*I confirm that this work is my own and is the final version. I have acknowledged each use of the words or ideas of another person, whether written, oral or visual*."

Step 1 Introduction / Preparation

Read the two articles listed below. The articles are **not** examples of IB maths Explorations but appeared in a professional journal for American math teachers and describe in detail how teachers might engage students in the exploration of a particular mathematical problem. Both articles illustrate good writing about mathematics at an appropriate level for IB mathematics. We will discuss aspects of the articles in relation to expectations for the Exploration you will write for your IA.

- article 1: ***Thinking out of the Box … Problem*** click here for article
- article 2: ***Rugby and Mathematics – A Surprising Link Among Geometry, the Conics and Calculus*** click here for article

Step 2 Choosing a topic for your Exploration

This stage is very important and you need to make a serious effort and listen to your teacher's advice. Listed on this page and the next page are 200 possible topics. Browse through the list and do some quick research (perhaps using Wikipedia) on any topic that catches your interest. There will be a 2-week consultation phase during which you will discuss your topic ideas with your teacher. You must complete an **Exploration Proposal Form** (see ~~page 6~~ last page) for each of your ideas before meeting with your teacher. Some of the questions that need to be addressed include: **(1)** does the topic involve math at a suitable level for your IB maths course? ; **(2)** is the topic narrow enough so that it can be treated sufficiently in 12 to 20 pages? ; and **(3)** does the topic lend itself to demonstrating personal engagement involving independent thinking and creativity? Can you envision some way that you could apply something of your own – your own viewpoint, your own examples, your own models (conceptual or physical), your own questions & ideas, etc.

■ Your topic must be approved by your teacher by the given deadline ■

◆ 200 Exploration ideas/topics ◆

Algebra & Number Theory

Modular arithmetic
Chinese remainder theorem
Applications of complex numbers
Diophantine equations
General solution of a cubic equation
Odd perfect numbers
Patterns in Pascal's triangle
Factorable integers of the form *ak* + *b*
Pythagorean triples
Combinatorics – art of counting
Loci and complex numbers
Divisibility tests
Graphical representation of roots of complex numbers
Euler's identity: $e^{i\pi}+1=0$
Probabilistic number theory
Natural logarithms + complex numbers
Twin primes problem
Diophantine application: Cole numbers
Polar equations
Palindrome numbers
Random numbers
Inequalities & Fibonacci numbers
Magic squares & cubes
Roots of unity
Complex numbers & transformations
Goldbach's conjecture
Fermat's last theorem
Continued fractions
Hypercomplex numbers
Applications of logarithms
Euclidean algorithm for GCF
Finding prime numbers
Algebraic congruences
Mersenne primes
Boolean algebra
Matrices & Cramer's rule
Egyptian fractions

Calculus/Analysis & Functions

Mean Value theorem
Applications of power series
Fundamental theorem of calculus
Second order differential equations
Torricelli's trumpet (Gabriel's horn)
Newton's law of cooling
Brachistochrone (min.time) problem
l'Hopital's rule and evaluating limits
Integrating to infinity
Hyperbolic functions
The harmonic series
Torus – solid of revolution

Probability & Probability Distributions

Normal distribution and natural phenomena
Monte Carlo simulations
Insurance and calculating risks
Determination of π by probability
Bayes' theorem
The Monty Hall problem
Random walks
Poisson distribution and queues
Lotteries
The birthday paradox

Geometry

Non-Euclidean geometries	Cavalieri's principle	Packing 2D and 3D shapes
Ptolemy's theorem	Hexaflexagons	Heron's formula
Geodesic domes	Proofs of Pythagorean theorem	Tesseract – a 4D cube
Minimal surfaces & soap bubbles	Map projections	Penrose tiles
Tiling the plane – tessellations	Morley's theorem	Cycloid curve
Symmetries of spider webs	Fractal tilings	Euler line of a triangle
Fermat point - polygons & polyhedral	Pick's theorem & lattices	Conic sections
Properties of a regular pentagon	Nine-point circle	Regular polyhedral
Geometry of the catenary curve	Euler's formula for polyhedral	Stacking cannon balls
Eratosthenes' - earth's circumference	Ceva's theorem for triangles	Area of an ellipse
Constructing a cone from a circle	Conic sections as loci of points	Consecutive integral triangles
Mandelbrot set and fractal shapes	Curves of constant width	Sierpinksi triangle
Squaring the circle	Polyominoes	Reuleaux triangle
Architecture and trigonometry	Spherical geometry	

Statistics & Modelling

Logistic function & constrained growth	Modelling growth of tumours	Traffic flow
Modelling epidemics/spread of a virus	Correlation coefficients	Hypothesis testing
Modelling the shape of a bird's egg	Central limit theorem	Modelling radioactive decay
Modelling growth of computer power	Least squares regression	Regression to the mean
Modelling change in record performances for a sport		

Numerical Analysis

Methods for solving differential eqns	Linear programming	Fixed point iteration
Methods of approximating π	Applications of iteration	Newton's method
Estimating size of large crowds	Generating the number *e*	Descartes' rule of signs

Logic & Sets

Codes and ciphers	Set theory and different 'size' infinities	
Mathematical induction (strong)	Proof by contradiction	Proving a number is irrational

Topology & Networks

Knots	Steiner problem	Chinese postman problem
Travelling Salesman Problem	Königsberg bridge problem	Handshake problem
Möbius strip	Klein bottle	

Games & Game Theory

The prisoner's dilemma Sudoku Gambler's fallacy Card games Knight's tour in chess

Physical, Biological & Social Sciences

Radiocarbon dating	Gravity, orbits & escape velocity	Biostatistics
Mathematical methods in economics	Genetics	Crystallography
Computing centres of mass	Elliptical orbits	Predicting an eclipse
Logarithmic scales-decibel, Richter, etc	Change in BMI for a person over time	
Fibonacci sequence and spirals in nature	Concepts of equilibrium in economics	

Miscellaneous

Paper folding	Designing bridges	Mathematical card tricks
Methods of approximating π	Barcodes	Applications of parabolas
Curry's paradox – 'missing' square	Voting systems	Terminal velocity
Music – notes, pitches, scales, etc	Towers of Hanoi puzzle	Photography
Flatland by Edwin Abbott (book)	Art of M.C. Escher	Harmonic mean
Sundials	Navigational systems	*A Beautiful Mind* (film)
The abacus	Construction of calendars	Slide rules
Different number systems	Mathematics of juggling	Airline routes

Step 3 Write a complete draft

Before you start writing your Exploration be sure to carefully read through the details for all five of the assessment criteria at the end of this guide. Along with a brief description and achievement level descriptors, there is also helpful *further guidance* notes for each criterion.

A draft is not an abbreviated or incomplete version of your Exploration. It must be **complete** – including an introduction, a conclusion, and a bibliography – with sufficient content to address your stated objective(s) and be in the range of 12 to 20 pages (double spacing, font Times New Roman). Your Exploration needs to be logically organized; use appropriate mathematical terminology and notation; include explanatory diagrams, graphs, tables, etc; contain citations to indicate where a source is used; and focuses on relevant mathematics. It is important to include your own thoughts, questions, reflections & ideas when possible. Although not required, it is recommended that you write in the first person; for example, "**I** decided that the best method is _____ because **I** realized that …"

Although the Exploration is an individual assignment and all the work must be your own, you are strongly encouraged to regularly consult with your teacher. Your teacher can provide verbal guidance and feedback while you are writing your draft.

Step 4 Teacher feedback

Your teacher will provide written feedback on the draft of your Exploration. Be sure to ask questions about any comments / feedback that you do not completely understand.

Step 5 Submit final version of your Exploration

From the time you receive written feedback on your draft you will have about ~~2~~ 6 weeks to revise your draft and complete the final version of your Exploration. Before submitting your final version, complete the student checklist on the next page →

The Exploration – Top Tips

1. Choose a topic in consultation with your teacher that: (i) you're interested in; (ii) involves math at a suitable level; (iii) is narrow enough for 12-20 pages; and (iv) has opportunities for personal engagement.
2. Your Exploration must have a clear aim (objective) that involves doing some mathematics. It is important to maintain a focus on the overall aim/objective and a focus on mathematical concepts and methods.
3. Although all the work on your Exploration must be your own, do not hesitate to ask your teacher for advice and feedback at any stage. Your teacher will provide written feedback on your complete draft.
4. Be sure you fully understand the expectations of the five assessment criteria (pg. 8-12), and refer to them while you are planning and writing your Exploration.
   [**Note**: Except for Criterion E (Use of Mathematics), the criteria descriptors are the same for SL and HL]
5. The Exploration is an opportunity to complete a significant assessment item (20% of IB score) while not under the pressure of timed exam conditions. Take advantage of the opportunity by following instructions, meeting deadlines, engaging & reflecting in your own way, and enjoying some math you are interested in.

Mathematics Exploration ☑ Student Checklist

1. Is your exploration written entirely by yourself – during which you avoided simply replicating work and ideas from sources you found during your research? ☐ Yes ☐ No
2. Have you strived to apply your personal interest; develop your own ideas; and use critical thinking skills when writing your exploration? ☐ Yes ☐ No
3. Does your exploration focus on good mathematical communication – and read like an article for a mathematical journal? ☐ Yes ☐ No
4. Have you included an introduction and conclusion, and clearly stated your aim? ☐ Yes ☐ No
5. Have you documented all your source material in a bibliography and included citations at the location in the paper where a source is used? ☐ Yes ☐ No
6. Not including the bibliography and any appendix, is your exploration 12 to 20 pages? ☐ Yes ☐ No
7. Are graphs, tables and diagrams sufficiently described and labelled? ☐ Yes ☐ No
8. Have you used and demonstrated mathematics that is at the same, or similar, level of the mathematics in your IB mathematics course? ☐ Yes ☐ No
9. Have you attempted to discuss mathematical ideas, and use mathematics, with a sufficient level of knowledge and understanding demonstrated on your part? ☐ Yes ☐ No
10. Are formulae, graphs, tables and diagrams in the main body of text? (large data sets, additional graphs, and large tables should be put in an appendix) ☐ Yes ☐ No
11. Have you used technology – such as a GDC, spreadsheet, mathematics software, drawing or word-processing software – to enhance mathematical communication? ☐ Yes ☐ No
12. Have you used appropriate mathematical language (notation, symbols, terminology) and defined key terms? ☐ Yes ☐ No
13. Is your Exploration double-spaced in Times New Roman font size 12? ☐ Yes ☐ No
14. Have you included – throughout your exploration – reflective and explanatory comments about the mathematics being explored and any results obtained? ☐ Yes ☐ No

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Criteria for the Exploration (IA)

Criterion A: Presentation (4 marks)

This criterion assesses the organization, coherence and conciseness of the exploration.

Achievement Level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

◘ Further Guidance ◘

A **well-organized** exploration includes an introduction, states the clear aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. If necessary, appendices can be used to include tables displaying large data sets, and additional graphs and tables that are not suitable for including in the main body of the paper because they will detract from its clarity.

A **coherent** exploration is logically developed, easy to follow ('reads well') and achieves its aim. This refers to the overall structure, including introduction, body, conclusion and how well the different parts link to each other. It is not necessary to give each part (section) of the paper a formal title.
A **concise** exploration does not include irrelevant material or unnecessary repetitive calculations, graphs or descriptions.
The use of technology is not required but it is encouraged when the choice of technology is appropriate and supports the development of the exploration. However, the use of analytic approaches (i.e. manual working) rather than technological ones does not necessarily mean there is a lack of conciseness.

Criterion B: Mathematical Communication (4 marks)

This criterion assesses to what extent the exploration clearly and effectively uses multiple forms of mathematical representation such as formulae, diagrams, tables, graphs and models.

Achievement Level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

◘ Further Guidance ◘

Mathematical communication includes the following:
• Using appropriate mathematical language (**notation, symbols, terminology**). [Note: Calculator and computer notation is only acceptable if it is software generated. Otherwise, it is expected that students use appropriate notation - preferably composed using an equation editor with word processing software.]
• Defining **key terms** and variables, where required.
• Using **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
• Employ a **deductive method** and set out proofs logically where appropriate.
If graphs are not labelled, computer notation is used and no other forms of mathematical communication then a score of 1 is likely be awarded. It is best to use multiple forms of representation, but it is possible to earn a score of 4 using one form of representation if it enhances the exploration and furthers its progress.

Criterion C: Personal Engagement (3 marks)

This criterion assesses the extent to which the student engages with the topic, expresses ideas in an individual way and does their best to "make the exploration their own". It is not a measure of effort.

Achievement Level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

◘ Further Guidance ◘

Students can demonstrate personal engagement in different ways. These include commenting on their own decisions, thinking independently and creatively, presenting mathematical ideas in their own way, making and testing predictions, and exploring the topic from different perspectives. There must be evidence of personal engagement demonstrated explicitly in the student's written work.
Textbook style explorations or reproduction of readily available mathematics without the student's own perspective are unlikely to achieve higher scores for this criterion. If it is necessary to include mathematical

working from a source such as a textbook, then the student must strive to explain and comment on the working in their own words as much as possible. Additional ways of showing personal engagement include: investigating their own questions and conjectures; creating their own examples; presenting ideas and results in their own words; creating their own models or functions.
Significant personal engagement: The student demonstrates authentic personal engagement in the exploration on several occasions and it is evident that these drive the exploration forward and help the reader to better understand the development and the aim of the exploration.
Outstanding personal engagement**:** The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the exploration topic and the reader better understand the development and the aim of the exploration.

Criterion D: Reflection (3 marks)

This criterion assesses how well a student reviews, analyses and evaluates different aspects of their exploration and, in particular, any results that are obtained.

Achievement Level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

◘ Further Guidance ◘

Although reflection should occur in the conclusion, a student should strive to include reflection throughout the exploration.
Simply describing results represents **limited reflection**. Further consideration and genuine analysis are required to achieve higher scores for this criterion.
Some ways of showing **meaningful reflection** are: linking to the aim of the exploration, commenting on what has been learned, considering limitations of methods or results, and comparing different mathematical approaches.
Critical reflection is deeply insightful and directly relevant to the exploration's aim. It supports the development of the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: commenting on the appropriateness of different approaches, discussing implications of results, discussing strengths and weaknesses of techniques, and making links between different areas of mathematics.
Substantial evidence is critical reflection that occurs throughout the exploration and not just at the end of the exploration. If reflection appears only at the end of the exploration, then to achieve a score of 3 the student's reflections must be of a very high quality and clearly indicate how the student's thoughts and ideas contributed to the development of the exploration.

Criterion E: Use of Mathematics (6 marks)

* ***Criterion E descriptors are <u>not</u> the same for SL and HL*** *

This criterion assesses to what extent you use mathematics that is relevant to your Exploration.

Criterion E: Use of Mathematics – HL

Achievement Level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

◘ Further Guidance HL ◘

Relevant mathematics supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is **commensurate with the level** of the course, which means that the mathematics explored should either be in the SL syllabus, or at a similar level or slightly beyond. However, mathematics at a level beyond the syllabus is **not** required to achieve a higher score for this criterion.

A key word in the descriptor is '**demonstrated**' which means "to make clear by reasoning or evidence, illustrating with examples or practical application". Answers to mathematical working must be accompanied by explanation or justification in order to demonstrate understanding.

For knowledge and understanding to be **thorough** it must be demonstrated throughout the exploration. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.
The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate but **understanding must be demonstrated** for the student to earn a score higher than 1. For example, merely substituting values into a formula does not necessarily demonstrate understanding of the mathematics.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single syllabus item. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and the student clearly demonstrates understanding of the mathematics used, then a high score can be earned for this criterion.

Precise mathematics is error-free and always uses an appropriate level of accuracy.

Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics. If the level of mathematics is only at the SL level, to be considered 'sophisticated' it needs to be used in a complex way that is beyond what could reasonably be expected of an SL student.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

On the first Monday of class, August 11th, 2025,
you will SHARE A GOOGLE DOC with MS. WHITE via Canvas,
which will allow her to share feedback regarding your IA Topic.

Your answers to the following questions do not need to be polished or formal. Just provide an explanation of what you plan to explore. IFF you have already begun a rough draft, please insert whatever you have, or share that document instead.

Please include *your* name in the document name like this: SmithRebeccaMathIAPlan.docx

IB Math A&A IA Proposal/Plan

- What is the central aim/objective/question of your IA?
- Why did you choose this topic?
- Provide any relevant background information.
- What mathematical skills/concepts/methods will you use?
- What resources have you consulted?